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**Iannis Xenakis' Contributions
to 20th Century Atonal Harmony
and to the History of Equal Temperament**

The Aristoxenean Sieve Theory, and the Ptolemaic Alternative
*Some Thoughts on Our Perception of Musical Intervals*¹

In his influential orchestral piece *Metastaseis* (1953-54), Iannis Xenakis introduced a number of great new sonic entities, exciting innovations that radically expanded the vocabulary of music. I would like to mention just two of them here: complex glissando textures (with up to 46 different simultaneous glissando speeds) and dense chromatic or micro-chromatic tone clusters. These are major contributions to 20th century atonal harmony, because they serve to neutralize our ear's refined capability of *perceiving harmonic relations* by creating the impression of noise (which is generated by an accumulation of masses of tempered pitched sounds).

In the very next piece, *Pithoprakta* (1955-56), Iannis Xenakis organized his sound masses by introducing the notion of probability, and here the basic musical parameter of pitch "is important only in the global sense of conveying registral boundaries (e.g. high versus low register, wide versus narrow range, fixed versus evolving placement)."² Here the pitches are no longer composed according to any kind of either tonal or atonal harmonic counterpoint; instead they are determined by a probability algorithm defined at a higher structural level.

I think that György Ligeti was very impressed and influenced by Xenakis' new polyphonic musical textures. In 1962 he presented and analyzed *Metastaseis* and *Pithoprakta* in his lecture at the International Summer Courses for New Music in Darmstadt. When I studied with him about twelve years later, however, he criticized Xenakis for "not having a sense of harmony", referring to these pieces. Like Witold Lutoslawski, Luigi Nono and others, György Ligeti was interested in developing and reintroducing some harmonic principles to overcome the "impasse of the

¹ This essay is a free reconstruction of a talk on January 30, 2011 at the CEIAT festival 'Iannis Xenakis in Los Angeles' in the Museum of Contemporary Art.

² James Harley. *Xenakis : His Life in Music* (New York and London: Routledge, 2004) 14.

chromatic scale”, as Xenakis called it;³ and I believe that Ligeti wasn’t yet aware of the fact that Xenakis had also been working on solving the same problem – in his own unique and profound way – at the same time since the mid-1960s. Xenakis began by first studying Aristoxenos’ *Elementa Harmonica* to analyze the structure of Ancient Greek Music, and from this basis he then devised his “Sieve Theory”,⁴ which became a powerful compositional tool in the creation of much of his music since the mid-1960s. I regard this invention as another substantial contribution to the methods of 20th century atonal harmony.

It would be exciting to analyze Xenakis’ scores in order to assemble a comprehensive compilation of all the various pitch sieves (or scales) that he used in his music – the remarkable counterpoint of the postlude in *Nomos Alpha*, the bright and diatonic introduction of *Jonchaeis*, or the stunning fireworks of *Ata*. In this essay, I will discuss Xenakis’ Sieve Theory in a more general way, focusing on the historical relevance of his invention. I believe that it was a major step, perhaps even the final step towards what I’d like to call the *historical fulfillment of our Twelve-Tone-System of Equal Temperament*.

This can perhaps be explained by a quick glance at the function and the history of this tuning system. In Western music, Equal Temperament was first devised as a standard tuning system for fretted instruments only. The lutenist and composer, Vincenzo Galilei (circa 1520-1591), the father of the astronomer Galileo Galilei, calculated and defined the size of the tempered semitone as a practical approximation to the string length ratio 18:17 (circa 99 cents), which was “convenient for fretting the lute”.⁵ But there was a general consensus amongst musicians during the 16th and 17th century that this rather dissonant temperament with its harsh major thirds and sixths is inferior to Meantone Temperament and not an appropriate and desirable tuning system for keyboard instruments like the organ and harpsichord.⁶ As the creative urge to also explore the more distant keys had become increasingly irresistible, experimental keyboard instruments with split upper keys on multiple registers were built in Italy to complete a circular modulating Meantone System (with as many as 31

³ Bálint András Varga. *Conversations with Iannis Xenakis* (London: Faber and Faber, 1996) 93, 94, 96.

⁴ Iannis Xenakis. *Formalized Music: Thought and Mathematics in Composition* (Hillsdale, NY: Pendragon Press, 1992), Chapter VII: *Towards a Metamusical*.

⁵ J. M. Barbour. *Tuning and Temperament: A Historical Survey* (East Lansing: Michigan State College Press, 1951; Mineola, N.Y.: Dover Publications, 2004) 8, 57; source text: Vincenzo Galilei. *Dialogo della musica antica et della moderna* (Florence, 1574).

⁶ This is clearly evident from history itself. For what other reasons could there have been for restricting the use of Equal Temperament to the fretted lutes and guitars? See also: Mark Lindley. *Temperaments*, *New Grove Dictionary of Music and Musicians* (London: Macmillan Publishers Limited, 1980) volume 18, 664-666.

tones in the octave), in which the tempered whole-tone (i.e. the meantone) was divided into five ‘dieses’. But the technical difficulties of building, maintaining, tuning, and playing such fantastic instruments kept them as exceptions, and they could not enter into common practice. By the end of the 17th century, musicians therefore began to experiment with many different modifications of the traditional Meantone Temperament, and these irregular keyboard temperaments with all their expressive microtonal diversity became highly popular. Johann Sebastian Bach’s own favorite irregular keyboard temperament, which has recently been discovered (right on the top of Bach’s handwritten title page of *The Well-Tempered Clavier*) by Bradley Lehman,⁷ was, as it seems, the very most subtle and refined attempt to represent all twenty-four major and minor keys on a standard keyboard with only twelve pitches within the octave, while also maintaining a complete compatibility with the tone system of Extended Meantone Temperament, which was common practice for non-keyboard instruments throughout the 18th century.

Like his contemporaries, Wolfgang Amadeus Mozart thought in terms of Extended Meantone Temperament until the end of his life, differentiating between the *small chromatic semitone* (i.e. the difference between a major and a minor third, like d-d[#]) and the *large diatonic semitone* (i.e. the difference between a fourth and a major third, like d-e^b), and thus between at least nineteen different pitches per octave: c-c[#]-d^b-d-d[#]-e^b-e-e[#]-f-f[#]-g^b-g-g[#]-a^b-a-a[#]-b^b-b-b[#]-c.⁸

When Jean-Philippe Rameau changed his opinion and started advocating Equal Temperament in 1737,⁹ he triggered a vivid discussion about its pros and cons in France, but the Broadwood piano company in London did not adopt Equal Temperament until 1846.¹⁰ Once the discussions about the sound of this keyboard temperament had finally subsided, it did not take long before it was in fact regarded as the “new modern tone system”. Given the immense popularity of the piano and its central role in our music education, even the theory of tonal harmony was eventually understood in terms of Equal Temperament. The acoustical explanations and critical comments published in 1863 by the great physicist Herman von Helmholtz in his pioneering book *On the Sensations of Tone as a*

⁷ Bradley Lehman. “Bach’s extraordinary temperament: our Rosetta Stone” in *Early Music*, volume XXXIII No. 1+2 (Oxford University Press, Feb. and May 2005).

⁸ See the papers of Mozart’s pupil Thomas Attwood, published as part of the new Mozart Edition, series X/30 (Kassel: Bärenreiter, 1969).

⁹ Jean-Philippe Rameau. *Génération Harmonique* (Paris, 1737); English translation: Deborah Hayes. *Rameau’s “Theory of Harmonic Generation”: An Annotated Translation and Commentary of “Génération Harmonique” by Jean-Philippe Rameau* (Stanford, 1968).

¹⁰ Ross W. Duffin. *How Equal Temperament Ruined Harmony (and Why You Should Care)* (New York and London: W.W.Norton & Company, 2007) 106.

*Physiological Basis for the Theory of Music*¹¹ did not have an immediate impact on the general school of thought amongst musicians.

But the major German conductor, Hans von Bülow (1830-1894), a close friend of Wagner, Liszt and Brahms, used to call this new tone system of Equal Temperament “the piano lie”¹² all his life. And in 1911 the eminent thinker, Arnold Schoenberg, commenced his forward-looking *Theory of Harmony* with a discussion of the *harmonic series of the overtones*. In this context he called the standard tone system of Equal Temperament “a compromise between the natural intervals and our inability to use them” and “a truce made for an indefinite period of time”.¹³ Only ten years later, he invented his dodecaphonic method for an atonal pitch organization that is conceptually based on the specific virtue of the equal-tempered twelve-tone system, i.e. its unrestricted “transposability”.¹⁴ The atonal twelve-tone music, as well as the serial music of the 1950s, was literally composed *for* this tone system, and it may therefore be regarded as commencing its historical fulfillment. For the first time in its long history, Equal Temperament was not a compromise but the perfectly adequate tuning system for this new kind of music.

Another conceptual reference to the tempered twelve-tone system with its equal semitones was achieved around the same time during these early 1920s in the atonal music of Edgard Varèse, which fully exploited the dissonant sonority of stacked equally-sized semitones, – and later in the “cluster music” developed during the late 1950s and early 1960s by Iannis Xenakis, Luigi Nono, György Ligeti, Krzysztof Penderecki, and others.¹⁵ Yet another remarkable summit in the history of Equal Temperament was established by its microtonal extension in the work of Alois Haba and Julian Carillo, and by Ivan Wyschnegradsky, who devised

¹¹ Herman von Helmholtz. *Die Lehre von den Tonempfindungen als physiologische Grundlage für die Theorie der Musik* (Braunschweig, 1863); English translation by Alexander Ellis. *On the Sensations of Tone as a Physiological Basis for the Theory of Music*, translated from the edition of 1877 (New York: Dover Publications, 1954).

¹² Anonymous. *Testimonials, Etc. (Translations) Relating to the “Enharmonium,” Invented by Shohé Tanaka*. London, 1891 (The Virtual Laboratory, Max-Planck-Institute for the History of Science, Berlin, ISSN 1866-4784 - <http://vlp.mpiwg-berlin.mpg.de>).

¹³ Arnold Schönberg. *Harmonielehre* (Vienna, Universal Edition, 2nd edition, 1922); English translation by Roy E. Carter (Berkeley: University of California Press, 1978).

¹⁴ In the tone system of Equal Temperament, any pitch collection (e.g. motif, chord, or twelve-tone-row) can be precisely, i.e. literally transposed. This is not generally possible in other tuning systems (e.g. Just Intonation, Meantone Temperaments, or irregular keyboard temperaments, like that of Francesco Antonio Vallotti, for example).

¹⁵ For example, Iannis Xenakis: *Metastaseis*, *Pithoprakta*, and often again much later, e.g. in *Ata*; Luigi Nono: *Canti di vita e d'amore*; György Ligeti: *Apparitions, Atmosphères, Requiem*; Krzysztof Penderecki: *Anaklasis*, *Threnody to the Victims of Hiroshima*, *Fluorescences*.

“ultra-chromatic scales,” as he called them (e.g. various sequences of equal-tempered quartertones and semitones),¹⁶ and thus employed an equivalent of the Sieve Theory before Xenakis had even formulated it.

To end my summary of the historical context of Iannis Xenakis’ Sieve Theory, I would like to mention two unique pieces by James Tenney from 1985.¹⁷ Written for pedal harps and guitars, i.e. for those non-keyboard instruments that are most intimately tied up with the tuning system of Equal Temperament, these two compositions are profound investigations into the harmonic potential of the tempered “72-set”, as James Tenney called it, which divides the octave into 72 discrete equidistant pitches. Using the 72 pitches of this “twelfth-tone sieve” with the aim of better approximating as many *just intervals* as possible, he was already working with a very different mind set than Iannis Xenakis – looking at Equal Temperament; not any more from *within*, but from *above* or *beyond*, so to speak, as a protagonist of next generation *tonal thought*, once again conscious of harmonic relations between the musical tones. This is an important distinction, and it shall be developed as we proceed.

Iannis Xenakis was fully aware of the conceptual bond between his Sieve Theory and the tone system of Equal Temperament, which was generally taken for granted in its heyday fifty years ago – in fact, just before the much more relaxed sound of Meantone Temperament was rediscovered. In his article *Towards a Metamusic*, in which he first presented his Sieve Theory in 1967, Iannis Xenakis says:

[...] the tempered diatonic system—our musical terra firma on which all our music is founded—seems not to have been breached either by reflection or by music itself. This is where the next stage will come. The exploration and transformations of this system will herald a new and immensely promising era.¹⁸

We can sense the energy of his optimistic belief in the remaining musical potential of the tempered tone system that was yet to be exploited. There is no trace any more of the profound, fundamental skepticism Arnold Schoenberg had articulated about half a century earlier, before he proceeded on the basis of his courageous decision to fully embrace the

¹⁶ Charles Amirkhonian. *Interview with Ivan Wyschnegradsky*, recorded 1976 (radiom, otherminds.org) <http://radiom.org/detail.php?et=interview&omid=AM.1976.06.04>.

¹⁷ James Tenney. *Changes: Sixty-four Studies for Six Harps* (1985) and: *Water on the Mountain... Fire in Heaven* for 6 electric guitars (1985); see also: James Tenney. “About *Changes: Sixty-four Studies for Six Harps*” (*Perspectives of New Music* 25, 1987).

¹⁸ Xenakis. Op. cit. 182.

tempered system anyhow, by composing the dodecaphonic atonal music it asked for.

In order to introduce his Sieve Theory, Iannis Xenakis first proposes in *Towards a Metamusic* the valuable new categories called “outside-time”, “in-time”, and “temporal”, and then gives a comprehensive account of the outside-time structure of Hellenic and Byzantine music. He follows the ancient Greek source texts (mainly the brilliant above-mentioned treatise *Elementa Harmonica* by Aristoxenos), albeit with a radically formalized modern terminology, which is entirely based on Equal Temperament by using the twelfth-tones of the 72-set (which Xenakis calls “Aristoxenean segments”) as a general unit of measurement.

In fact, Aristoxenos briefly touches upon this twelfth-tone in his treatise in order to define the small difference between the third-tone (*chromatic diesis*) and the quarter-tone (*enharmonic diesis*). Aristoxenos discusses the magnitudes of the intervals between the four notes of the *tetrachord* in the diatonic, chromatic and enharmonic genera or modes (with their various microtonal “shadings”) by specifying the possible pitch ranges for the two movable middle notes (*lichanos* and *parhypate*) of the *tetrachord meson*; and he immediately adds the remark:

For it must be understood that the *lichanoi* are unlimited in number. Wherever you arrest the voice in the range of the *lichanos* will be a *lichanos*, and no place in the *lichanos* range is empty or incapable of receiving a *lichanos*.¹⁹

This means that the melodic function of this second-highest note of the tetrachord still remains the same within the genus (or mode) when its intonation is slightly altered within its permissible pitch range. And somewhat later, in Book II, he explains:

Each of the genera moves with what perception apprehends as its own characteristic movement while using not just one division of the tetrachord, but many. Thus it is clear that the genus can remain constant while the magnitudes change, since up to a certain point the genus does not change when the magnitudes do, but remains the same: and while the genus remains constant it is reasonable to suppose that the functions [*dynameis*] of the notes do too.²⁰

We can imagine that the intonation of the two movable middle notes of the tetrachord was indeed performed with much greater flexibility than

¹⁹ Quoted from *Greek Musical Writings, Volume II, Harmonic and Acoustic Theory*, edited by Andrew Barker (Cambridge, New York, Melbourne: Cambridge University Press, 1989) 144.

²⁰ *Ibid.* 162-163.

the schematic representation of the note positions in Xenakis' summary suggests – whereas the pitches of the fixed outer notes were certainly not tempered at all, but tuned in perfect fourths, fifths, unisons and octaves. Regarding the intonation of consonances and dissonances, Aristoxenos says near the end of Book II:

Of the magnitudes of intervals, those of the concords appear to have either no range of variation [*topos*] at all, being determined to a single magnitude, or else a range which is quite indiscernible, whereas those of the discords possess this quality to a much smaller degree. Hence perception relies much more confidently on the magnitudes of concords than on those of discords.²¹

Aristoxenos was the son of a musician and a disciple of Aristotle, and in his revolutionary endeavor to write up the first full-fledged account of the “science concerned with melody” he avoided even the slightest reference to the impressive knowledge about the physics and the mathematics of sound accumulated by the previous generations of *Pythagoreans*. They had interpreted sound as the result of an impact and as movement of the air, studied the vibrations of strings at different speeds and analyzed the magical mathematical ratios between various string lengths and their associated pitches. Rejecting their theories and methods, especially their precise “language of the ratios” (as Harry Partch has called this powerful Pythagorean conceptual tool),²² as irrelevant or misleading “extraneous territory” for the practicing musicians, Aristoxenos introduced a new and very different set of practical musical concepts (some of which he must have adopted from the musicians who had taught him). By discussing musical intervals in terms of their sizes, i.e. by focusing the attention of the ear and the mind on the *melodic distances* between the notes, and not on their *harmonic relationships* like the Pythagoreans, Aristoxenos inaugurated a second school of thought amongst musicians. They were henceforth (and even to this day!) divided into these two opposing groups called the *Pythagoreans* and the *Aristoxeneans*.

With his fundamental dedication to Equal Temperament, Iannis Xenakis was of course a genuine Aristoxenean. Let us look at his opinion on this important topic of the two languages:

Attention must be drawn to the fact that he [Aristoxenos] makes use of the additive operation for the intervals, thus foreshadowing logarithms before their time; this contrasts with the practice of the Pythagoreans, who used the geometrical (exponential) language, which is multiplicative. Here the method of Aristoxenos is fundamental since: 1. it constitutes

²¹ Ibid. 168.

²² Harry Partch. *Genesis of a Music* (Madison: University of Wisconsin Press, 1949).

one of the two ways in which musical theory has been expressed over millennia; 2. by using addition it institutes a means of “calculation” that is more economical, simpler, and better suited to music; and 3. it lays the foundation of the tempered scale nearly twenty centuries before it was applied in Western Europe.

Over the centuries the two languages – arithmetic (operating by addition) and geometric (derived from the ratios of string lengths, and operating by multiplication) – have always intermingled and interpenetrated so as to create much useless confusion in the reckoning of intervals and consonances, and consequently in theories. In fact they are both expressions of group structure, having two non-identical operations; thus they have a formal equivalence.²³

I think that Xenakis’ reasons for preferring the Aristoxenean method are not that strong. This argument of “greatest convenience” has always been used by the advocates of Equal Temperament, but I believe that it does not suffice to secure the exclusive predominance of this tuning system in the long run. “This reduction of the natural relations to manageable ones cannot permanently impede the evolution of music; and the ear will have to attack the problems, because it is so disposed.” [original spacing maintained] says Arnold Schoenberg in 1911 in his *Theory of Harmony*,²⁴ and he follows this thought all the way, in his radical and visionary mind, saying:

Then our scale will be transformed into a higher order, as the church modes were transformed into major and minor modes. Whether there will then be quarter tones, eighth, third, or (as Busoni thinks) sixth tones, or whether we will move directly to a 53-tone scale ... we cannot foretell. Perhaps this new division of the octave will even be untempered and will not have much left over in common with our scale.

If we have an interest in developing new ways of dealing with the musical parameters of pitch, timbre and tonal relationships, we must sharpen our tools by combining the specific virtues of both languages now, completely avoiding the least bit of confusion. Let us reconsider the old Pythagorean method of reckoning intervals by their *frequency ratios*, which precisely specify not only their melodic magnitude, but at the same time, their harmonic complexity and their individual timbre which is characterized by their *partial unisons*, *combination tones* and *periodic signatures*. All of this useful data is represented by the two numbers making up each ratio. Perhaps we may indeed appreciate these resonant sounds and the wealth of subtle pitch inflections that will come about, if we fully embrace the phenomena of psychoacoustics and give up the

²³ Xenakis. Op. cit. 185.

²⁴ Schoenberg. Op. cit. (German Edition: 22, English translation: 25).

habit of tempering our intervals – by adopting the basic principle of *microtonal just intonation*. For what else could possibly have the substance and power to be capable of effectively overthrowing the reign of Equal Temperament? An exciting and alarming musical enrichment has been achieved during the past few decades by creative efforts to reveal the power and beauty of “noise music”. The sensation of noise arouses in us the somewhat frustrating awareness that our remarkable *capability of harmonic perception* can never be engaged by these sounds at all. But the ear and the brain have a natural, innate desire to perceive and process the pitches of musical tones, i.e. sounds with harmonic timbres.

As a modern Pythagorean acoustician, James Tenney has revitalized the language of the frequency ratios in the grand tradition of Helmholtz and Partch, all within the creative light of cutting-edge psychoacoustics. He supplied us with a universal conceptual model of our brain’s harmonic perception, which he called “harmonic space”.²⁵ I believe that this infinite multi-dimensional lattice representing all the non-tempered pitches is the new concept that can succeed our simplified model of the circle of twelve equally tempered fifths. All the musical tones in James Tenney’s matrix are precisely arranged according to their mutual harmonic relationships, according to the order suggested by the unisons of their partials. There is also a *pitch-height projection axis*, which represents the basilar membrane of the inner ear and runs at an angle of 45° to every axis right through James Tenney’s harmonic space; and upon this *glissando continuum* each non-tempered pitch, or pitch-point in harmonic space, may specify its precise frequency position.

Thus any number of microtonal scales or pitch collections can be filtered out of harmonic space. The intervals between adjacent notes will most likely have many different magnitudes. Moreover, it will usually not be the case that they are composite multiples of some general measurement unit. Like Harry Partch’s 43-note scale, these harmonic pitch collections display a subtle microtonal melodic order that resembles the irregular patterns of colored lines in the filtered spectrum of light – and not those on the ruler.²⁶

Let us return to Aristoxenos. His most influential new concept was that of the *semitone*, which must certainly have caused quite a stir among the Pythagoreans. For they had divided the octave (2:1) into a fourth and a fifth (4:3:2), which could then be subdivided into a minor and a major

²⁵ James Tenney: *John Cage and the Theory of Harmony*, first published in *SOUNDINGS 13: The Music of James Tenney* (Santa Fe: Soundings Press, 1984); also available at www.plainsound.org.

²⁶ E.g. the uniform intervals of 100 cents in Equal Temperament, or of some other “unit of displacement” in a Xenakian pitch sieve.

third (6:5:4), and so on; so they knew both from experience and reason that none of such tuned intervals with *superparticular ratios* can ever be harmonically divided into two or several equal parts. A real semitone can only exist in Equal Temperament, and we have seen that Aristoxenos understood his new technical term as a handy approximation. Facing the severe criticism of the Pythagoreans, he insisted on the assumption that the fourth is made up of two and a half tones – despite the well-known fact that the difference between the *fourth* (4:3) and the *ditone* (81:64), the so-called *leimma* (256:243), is somewhat smaller than half a tone.

Yes indeed, we can distinguish many different semitones; each of them has its own particular size – and its own particular harmonic function, identity or meaning. Besides the Pythagorean *leimma* (which has a size of 90 cents and is constructed out of five consecutive fourths or fifths), we have already encountered Mozart's *mezzo tuono grande*, the large diatonic semitone (16:15 = 112 cents), and Mozart's *mezzo tuono piccolo*, the small chromatic semitone (25:24 = 71 cents); but there are in fact more than twelve different semitones between the largest (15:14 = 119 cents) and the smallest (28:27 = 63 cents), and they may all be called a semitone. They can all be performed in non-tempered just intonation – within the appropriate musical setting that is informed by the mathematical know-how of Pythagorean acoustics.

To conclude, I'd like to quote a few words from the other great Hellenistic sage: Claudius Ptolemy (90-168 AD), whom Iannis Xenakis did not hold in high esteem.²⁷ This devoted Alexandrian mathematician, astronomer, geographer and music theorist must have had an extraordinary ear and musical feeling. In his enlightened treatise on music, which he called *Harmonics*, he says:

For in everything it is the proper task of the theoretical scientist to show that the works of nature are crafted with reason and with an orderly cause, and that nothing is produced by nature at random or just anyhow, especially in its most beautiful constructions, the kinds that belong to the more rational of the senses, sight and hearing. To this aim some people seem to have given no thought at all, devoting themselves to nothing but the use of manual techniques and the unadorned and irrational exercise of perception, while others have approached the objective too theoretically. These are, in particular, the Pythagoreans and the Aristoxeneans, and both are wrong. For the Pythagoreans did not follow the impressions of hearing even in those things where it is necessary for everyone to do so, and to the differences between sounds they attached ratios that were often inappropriate to the phenomena, so that they provided a slander to be directed at this sort of criterion by those whose opinions differed. The Aristoxeneans, by contrast, gave most

²⁷ Xenakis. Op. cit. 185.

weight to things grasped by perception, and misused reason as if it were incidental to the route, contrary both to reason itself and to the perceptual evidence [*to phainomenon*] – contrary to reason [*logos*] in that it is not the distinguishing features of sounds that they fit the numbers, that is, the images of the ratios [*logoî*], but to the intervals between them [...]²⁸

Iannis Xenakis' Sieve Theory serves to arrange the *distances* between the notes. Thus this method of composition is fundamentally related to that of the Serial Music, which Xenakis never liked. Both concepts establish an atonal pitch organization for the tone system of Equal Temperament, where the musical tones are not tuned according to their inner structure and according to their mutual tonal relationship, but in such a way that they are *equally far apart* from each other.

A few pages later, Ptolemy says:

This shows, then, that we should not find fault with the Pythagoreans in the matter of the discovery of the ratios in the concords, for here they are right, but in that of the investigation of their causes, which has led them astray from the objective; but we should find fault with the Aristoxeneans since they neither accepted these ratios as clearly established, nor, if they really lacked confidence in them, did they seek more satisfactory ones – assuming that they were genuinely committed to the theoretical study of music. For they must necessarily agree that such experiences come to the hearing from a relation that the notes have to one another, and further that where the impressions are the same, the differences are determinate and the same.²⁹

Ptolemy explains that the particular sensation is created by the specific timbre of an interval between two musical tones, by the harmonic quality of its tone relationship, i.e. by its overall compound sound which is made up of all the partials and combination tones. That is what generates the identity and thus allows the ear to understand the interval, even when it is being transposed. For a transposed interval vibrates at different frequencies and at another virtual fundamental, but the pattern of the interferences between these frequencies is recognizably the same. He continues:

Yet in what relation, for each species [of concord], the two notes that make it stand, they neither say nor enquire, but as if the notes themselves were bodiless and what lies between them were bodies, they compare only the intervals [*diastaseis*] belonging to the species, so as to appear to be doing something with number and reason. But the truth is

²⁸ Quoted from *Greek Musical Writings*, Op. cit. 278-279.

²⁹ Ibid. 293-294.

precisely the opposite. For in the first place they do not define in this way what each of the species is in itself (as when people ask what a tone is, we say that it is the difference between two notes that comprise an *epogdoic ratio* [i.e. 9:8]); instead, there is an immediate shift to yet another undefined item, as when they say that the tone is the difference between the fourth and the fifth [...] And if we enquire after the magnitude of the difference in question, they do not explain even this without reference to another, but would only say, perhaps, that it is two of those of which the fourth is five, and that this again is five of those of which the octave is twelve, and similarly for the rest, until they come back round to saying ‘...of which the tone is two’.³⁰

Ptolemy’s last remark sounds rather like a parody of our now historical, old-fashioned 20th century *tempered thought*, as displayed for example in the formalized Pitch-class Set Theory, which does not even have a proper name for the octave and calls a fourth the interval number “5”, the major third “4”, the tritone “6” – and so on. This terminology is of course perfectly adequate for the intervals of Equal Temperament, in which the tempered ditone, for example, is defined by the 12th root of the octave (2:1) to the power of 4. And also Iannis Xenakis’ idea of a general “unit of elementary displacement,”³¹ upon which his Sieve Theory is based, is of course a completely appropriate compositional tool for establishing a conceptual order within the set of equidistant notes available in any kind of Equal Temperament.

However, as Ptolemy says, the ear does not only perceive the magnitude of an interval between two musical tones (i.e. their *pitch distance*), but it also recognizes the timbre of their compound sound (i.e. their *harmonic distance*, as James Tenney called it).³² This psychoacoustic phenomenon is not taken into account on the basic conceptual level in either of these compositional methods. Nevertheless Arnold Schoenberg’s Twelve-Tone-Technique and Iannis Xenakis’ Sieve Theory have both come about out of historical necessity, for it is the very principle of Equal Temperament itself that is in fact subjecting the musical tones to this kind of mutual alienation.³³

³⁰ Ibid.

³¹ Xenakis. Op cit. 194-200.

³² Tenney. Op. cit.

³³ I thank Marc Sabat and Georgi Dimitrov for their invaluable help.