

Wolfgang von Schweinitz

# The Subharmonic Scale

INTONATION STUDY  
for two violins

op. 55

2010

*composed for*  
*Andrew Nathaniel McIntosh and Andrew Tholl*  
*and the*  
*Machine Project*  
*Hammer Museum, Los Angeles*

PLAINSOUND MUSIC EDITON

# Performance Notes

*The open strings are tuned in non-tempered perfect fifths. The piece is to be played without any frequency vibrato, but with a lot of bow.*

*The pitches are notated in the “Extended Helmholtz-Ellis JI Pitch Notation” (see legend below). The score also denotes the mathematical frequency ratios for most of the intervals as an additional tuning information. The numbers of these ratios are as it were the stenographic “code name” of the interval, specifying not only its size, but also the partial-unisonos and difference tones relevant for its timbre, as well as the root of the interval and its relative degree of consonance or complexity.*

*This short study in just intonation features a segment of the subharmonic series, the inversion of the overtone series. The pitches of the descending scale are defined by wavelengths (i.e. string lengths) with the ratios 8 : 9 : 10 : 11 : 12 : 13 : 14 : 15 : 16 : 17, which means that its melodic steps are gradually getting smaller and smaller, beginning with a major whole tone (204 cents) and ending with a 17-limit semitone (105 cents). They are executed by a glissando movement of the 1<sup>st</sup> finger which always slides across the same amount of space on the fingerboard for each step.*

*(Every glissando in the bass line played on the G string of the 2<sup>nd</sup> violin extends over the distance of 1/18 of the open string’s length, while the glissando in the part of the 1<sup>st</sup> violin, which always accompanies the bass voice in pure major thirds and sixths, covers 1/20 of the open string’s length in measure 3, 1/15 of the open string’s length in measures 5 – 13, 1/30 of the open string’s length in measure 15 and 17, and 2/45 of the open string’s length in measure 19 – 21.)*

*Each new note at the end of a glissando can be precisely tuned by ear to the pitch of an open string or one of its natural harmonics by optimizing the sonority of the double stop.*

*In measure 11 the 1<sup>st</sup> violinist must take care to avoid confusing the microtonally diminished fourth 13/10 with an augmented septimal major third 9/7, which is only 19 cents smaller; and the 2<sup>nd</sup> violinist must take care to avoid confusing the neutral ninth 13/6 with the 11-limit neutral ninth 11/5, which is only 26 cents larger – and somewhat more consonant – than the 13-limit interval.*

*The diminished fifth plus two octaves 17/3 in measure 19 and 21 is somewhat difficult to tune due to the weakness of the high natural harmonic, but it is in fact the most consonant diminished fifth augmented by two octaves. The final note B# in the part of the 1<sup>st</sup> violin is 13 cents lower than the pitch of a C tuned a perfect fourth above G; and the final note G# in the part of the 2<sup>nd</sup> violin (17/12 below the open D string) is 20.5 cents lower than the pitch of a G# tuned as a consonant septimal diminished fifth 7/5 below the open D string (and 14.5 cents higher than the pitch of an A-flat tuned as a septimal augmented fourth 10/7 below D).*

*The part of the 1<sup>st</sup> violin contains two notes (2<sup>nd</sup> note in measure 8 and 12) which are not tuned in double stops, but as a pure major sixth above the bass note of the 2<sup>nd</sup> violin; when then part of the 1<sup>st</sup> violin is rehearsed alone, these two notes can easily be tuned by singing them (ad lib. an octave lower) while playing the previous note.*

## PERFORMANCE DURATION *circa 2 minutes*

*This piece has been commissioned by the Machine Project 2010, Hammer Museum, Los Angeles. It is dedicated to Andrew Nathaniel McIntosh and Andrew Tholl.*

# ACCIDENTALS

*for Just Intonation*

## EXTENDED HELMHOLTZ-ELLIS JI PITCH NOTATION

The exact intonation of each pitch is written out by means of the following harmonically defined accidentals:

$\flat\flat$   $\flat$   $\natural$   $\sharp$   $\times$

Pythagorean series of non-tempered perfect fifths  
(based on the open strings : ... c g d a e ...)

$\flat$   $\natural$   $\sharp$   $\times$        $\flat\flat$   $\flat$   $\natural$   $\sharp$

Lowers / raises the pitch by a syntonic comma  
 $(81/80) = \text{circa } 21.5 \text{ cents}$

$\flat$   $\natural$   $\sharp$   $\times$        $\flat\flat$   $\flat$   $\natural$   $\sharp$

Lowers / raises the pitch by two syntonic commas  
 $(81/80) * (81/80) = \text{circa } 43.0 \text{ cents}$

$\flat$        $\natural$

Lowers / raises the pitch by a septimal comma  
 $(64/63) = \text{circa } 27.3 \text{ cent}$

$\flat\flat$  or  $\flat\flat$        $\sharp\sharp$  or  $\sharp\sharp$

Raises / lowers the pitch by two septimal commas  
 $(64/63) * (64/63) = \text{circa } 54.5 \text{ cents}$

$\flat$        $\sharp$

Raises / lowers the pitch by an 11-limit undecimal quarter-tone  
 $(33/32) = \text{circa } 53.3 \text{ cents}$

$\flat$        $\sharp$

Lowers / raises the pitch by a 13-limit tridecimal third-tone  
 $(27/26) = \text{circa } 65.3 \text{ cents}$

$\flat$        $\sharp$

Lowers / raises the pitch of the subsequent accidental by a  
17-limit schisma  $(16/17) * (16/15) = (256/255) = \text{circa } 6.8 \text{ cents}$

$\flat$        $\flat$

Raises / lowers the pitch of the subsequent accidental by a  
19-limit schisma  $(19/16) * (27/32) = (513/512) = \text{circa } 3.4 \text{ cents}$

$\sharp$        $\flat$

Raises / lowers the pitch of the subsequent accidental by a 23-limit  
comma  $(23/16) * (8/9) * (8/9) * (8/9) = (736/729) = \text{circa } 16.5 \text{ cents}$

The attached arrows denoting the pitch alteration by a syntonic comma are transcriptions of the notation that Hermann von Helmholtz used in his book "Die Lehre von den Tonempfindungen als physiologische Grundlage für die Theorie der Musik" (1863). The annotated English translation "On the Sensations of Tone as a Physiological Basis for the Theory of Music" (1875/1885) is by Alexander J. Ellis, who refined the definition of pitch within the 12-tone system of Equal Temperament by introducing a division of the octave into 1200 cents. – The accidental for the pitch alteration by a septimal comma was devised by Guisepppe Tartini (1692-1770), the composer-violinist and researcher who first studied the production of difference tones by means of tuned double-stops.

# The Subharmonic Scale

## INTONATION STUDY

for two violins

ca. 56 *rubato ad lib. e sempre non vibrato*

Wolfgang von Schweinitz  
op. 55 (2010)

*con sord. ad lib.*

Violin 1

Violin 2

*f* *dolce* *sonore*

III 1 4/3 5/4 5/3

IV 1 3/2 8/3

III *f* *dolce* *sonore*

3 9:8 (-204 c) 10/3 5/2 10/3 13/4 10:9 (-182 c)

6 11:10 (-165 c) 11/5 45:44 (-39 c) 11/10

9 12:11 (-151 c) 6/5 5/2 10:9 (-182 c) 13:12 (-139 c) 13/10

13:12 (-139 c) 13/10 3 13:6 13/4

2

12

3 : 4

40 : 39 (-44 c)

14 : 13 (-128 c)

7 / 5

4

7 / 5

2

7 / 5

15

15 : 14 (-119 c)

3 / 2

3 / 2

9 / 8

16 : 15 (-112 c)

6 / 5

15 : 14 (-119 c)

16 : 15 (-112 c)

18

4 / 3

16 / 3

17 / 3

16 / 3

17 : 16 (-105 c)

16 : 17 (+105 c)

17 : 16 (-105 c)

16 : 17 (+105 c)

21

17 : 16 (-105 c)

17 : 16 (-105 c)

17 / 3

17 / 3

17 / 4

17 / 5

15 : 16

5 / 3

7 / 4

*ritardando*

*f*

*f*

circa 2 min.