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# SCALAR ORDER AS A COMPOSITIONAL RESOURCE

BEN JOHNSTON

When listening to music we hear changing sound qualities in rhythmic patterns which create an illusion of growth. Most people hear music most readily as rhythmic gesture. Their musical present moment is the beat, the bar, the phrase (or the equivalents of these in less traditional music).

Tone qualities, noise textures, and pitch combinations make up the musical "objects" which are composed into rhythmic gestures. These different kinds of phenomenal *Gestalts* are, physically speaking, different modes of vibration. Qualities and relationships of sounds are our way of perceiving great numbers of tiny events (vibrations) on a molecular scale. We cannot hear these individual events, but we can easily detect order in the patterns they make.[1]

The rhythmic gestures composed of these qualities and relationships are in turn composed into larger patterns. These larger contexts are not directly perceptible. Memory reconstructs images of them once we have heard them. Even as we listen to a musical composition for the first time, expectations can grow in us which result from an intuitive grasp of the larger design patterns of the music. The act of musical composition relates the order and pattern of sound vibrations to the order and pattern of musical shapes of larger duration. A composer makes the two interdependent by means of his construction and deployment of musical order on the scale of ordinary rhythmic perception.[2]

When we listen for *practical* purposes, we identify objects and actions by their sounds. This recognition depends upon a recognition of similar patterns of sound vibration. To listen *musically* is to turn one's attention to details of the sound patterns and to interrelations of these patterns on

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[1] Cf. Henri Bergson's *Matter and Memory* (authorized translation by Nancy Margaret Paul and W. Scott Palmer): London, George Allen and Unwin Ltd., p. 275: "In reality there is no one rhythm of duration; it is possible to imagine many different rhythms which, slower or faster, measure the degree of tension or relaxation of different kinds of consciousness, and thereby fix their respective places in the scale of being." Thus Bergson indicates the intimate connection between tempo and quality, or to be more accurate, between varying rates of development and the building up and relaxing of psychological tensions.

[2] For some of the ideas on rhythm and tempo in this paper I am indebted to Robert Erickson's "Time Relations," a lecture given at the 1962 Illinois Wesleyan Symposium in Bloomington, Illinois.

different time scales. For everyday needs we usually need only to compare the similarity or difference of sounds or at most to classify them loosely in terms of their qualities; for aesthetic purposes we need to understand relationships more precisely. We need as a basis an intuitive grasp of more sophisticated systems of relationship. The depth of musical understanding depends upon the ability to hear orderly relationships and upon the precision of interrelationships implicit in the various kinds of scalar order used to organize the sounds.

S. S. Stevens[3] presents four kinds of scales of measurement: nominal, ordinal, interval, and ratio. A nominal scale is a collection of equivalent and interchangeable items. An ordinal scale is a collection which is rank-ordered in terms of some attribute. An interval scale is a rank-ordered collection in which the intervals of difference between items are equal. A ratio scale is a rank-ordered collection in which the items are related by equal ratios. A meaningful conception of zero value of the attribute in question is necessary before such a scale can be formed. Each of these scales includes all of the measurement possibilities of its predecessors, plus one more. "Thus an interval scale can be erected only provided we have an operation for determining equality of intervals, for determining greater or less, and for determining equality (not greater and not less). To these operations must be added a method for ascertaining equality of ratios if a ratio scale is to be achieved." (Stevens, *Handbook of Experimental Psychology.*)

The assignment of letters to thematic sections in traditional musical analysis (e.g. ABA, ABACABA, etc.) is an example of analysis by nominal scale. We are here concerned with the determination of *same* and *different.* The conventional use of dynamic markings in music (*pp*, *p*, *mp*, *mf*, *f*, *ff*) is an example of ordering by ordinal scale, since it is in practice very difficult to decide what an "equal increment" of loudness is, and in musical scores all that is usually implied by *mf* is that it is louder than *mp* and not so loud as *f*. The *melodic* use of pitch (rather precisely stepped contours of pitch variation) is an example of ordering by interval scale. So also is the establishment of a regular metrical *beat* (recurring equal durations). The *harmonic* use of pitch (carefully tuned simultaneous pitch combinations) is an example of ratio scale ordering, as is, in general, the practice of tuning by ear.

The application made in this paper of Stevens' scale types is based upon musical usage rather than upon controlled psychophysical experiments. The assumption has been that much is to be gained by exploring

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[3] S. S. Stevens, "On the Theory of Scales of Measurement," *Science,* June 7, 1946, Vol. 103, No. 2684, pp. 677–680. See also S. S. Stevens, "Mathematics, Measurement, and Psychophysics" in *Handbook of Experimental Psychology,* S. S. Stevens (ed.), New York and London, John Wiley and Sons, Inc.

the nature of the audible order employed in musical composition, from the dual viewpoint of traditional conceptions of musical order and of psychoacoustical theory. Perhaps this and other similar investigations may suggest new lines of inquiry which can be undertaken by psychoacousticians. Certainly, it can add to the understanding of musicians to apply to their field discoveries in this related discipline.

If a system of scalar order can be grasped intuitively, patterns easy to remember can be composed by rearranging it. The psychological tension between a particular scalar order and the pattern imposed upon it by composition stimulates attention, memory, and interest. The interrelation of many such patterns stimulates associations: memories and images having similar patterns. Whether I experience these associations as sensory images, as emotional affects,[4] or as abstract patterns, the music has *meaning* because of them.

A pitch scale is a complex system of ordered relationships. It is not merely one kind of scalar order but a combination of two different kinds: melodic and harmonic scalar order.[5] I can make patterns of melody or harmony by using pitches according to rules of composition which are based upon rearranging these two kinds of scalar order. I may choose not to use both kinds of pitch order, or even not to use either kind. But the order systems possible for pitches are greatly more refined than those possible for non-pitches, because the physical patterns of vibration that make up pitch are much more orderly. With pitched sounds, the possibility for interrelation is perceptibly more precise and more varied.

An extreme case of non-pitched sound is *noise.* If the vibrations of a sound have no discoverable recurrence of pattern, then it is classed as a noise. I can classify noises as similar and different, or compare them in terms of some sound quality such as loudness, "texture," or general pitch region. For more elaborate comparisons and relationships between sounds, I need to perceive a regularity of vibration pattern. Pitch provides this: a regular periodic pattern dominates a pitched sound. If this pattern is too simply repetitive, the sound is not only "artificial" sounding but also more difficult to relate to other sounds.[6] Most musical tones have a harmonious complexity. Acoustically, pitches are tempos of vibration. A *harmonious* sound is caused by an ensemble of tempos of vibration which measure off simple proportions of duration. Harmonious-

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[4] For a thorough but one-sided discussion of the emotional imagery of music, cf. Suzanne Langer's *Feeling and Form,* New York, Charles Scribner's Sons, 1953.

[5] Stevens, "On the Theory of Scales of Measurement." Stevens' *interval* and *ratio* scale types correspond to *melodic* and *harmonic* scales.

[6] Analogously, a jigsaw puzzle, if all the pieces are congruent and regularly shaped, is much harder to put together than if less similar shapes are used.

ness, or consonance, is the experience of simple proportionality in vibration patterns, whether of a single complex sound or of combinations of simultaneous sounds.[7]

Almost all cultures intuitively divide the pitch continuum into *octaves.* An octave, acoustically speaking, is the relation between a given tempo of vibrations and the tempo which is *double* or *half* of it (mathematically speaking, in the ratio 2:1). Tones an octave apart seem, apparently independently of cultural conditioning, the "same" tone transposed to different pitch levels. For this reason, music uses the octave cyclically, as the basis of scale formation. Octaves are then divided by a scale of smaller intervals. The two conflicting criteria which condition this are simplicity and symmetry: that is, a preference for simplicity or consonance of harmonic pitch ratios, and a preference for dividing melodic intervals symmetrically, into "equal" smaller intervals.

Melodically, the pitch dimension is a linear succession of octaves, each internally subdivided into smaller intervals which we speak of adding and subtracting. But this view of pitch offers no explanation of the common harmonic experience of a gradual scale of consonance and dissonance. Listening harmonically to pitch intervals we are actually comparing tempos of vibration. Thus what we commonly call "adding" adjacent intervals is, in acoustics, multiplying their vibration ratios, and subtracting them is dividing their ratios. When we "divide" an interval into "equal" smaller intervals, we seek a smaller ratio which, when multiplied by itself a given number of times, equals the larger ratio: that is, we are extracting roots. This "division" produces "irrational" pitch ratios which seem dissonant or *out of tune* when compared to near equivalents which are simple ratios. (This procedure is the basis of equal temperament.)

Harmonic listening is too easy and too basic to be ignored, even in purely melodic music. Yet melodic preference for equal scale intervals is also strong. If a scale is derived harmonically, it must consist of intervals whose melodic sizes differ by what seems a negligible amount. What seems negligible depends mostly upon relative interval sizes but also upon cultural conditioning and upon "how good an ear" an individual listener has.

Different cultures have evolved different solutions to this problem. Western European music has evolved a melodic scale of twelve "equal" intervals per octave. This pitch distribution originated from a rationally derived scale. Historically, equal temperament (which is a melodic equalization of the scale intervals) gradually gained general acceptance

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[7] Cf. Helmholtz, *On the Sensations of Tone,* transl. by A. Alexander J. Ellis, New York, Dover.

during the eighteenth century, because it facilitated instrumental design and performance, particularly of keyboard instruments.[8] The selection of *twelve* notes per octave is due to the nature of rationally derived scales and to musical practices resulting from their use (cf. a thorough discussion of pitch ratio scales, below). Rational scales with other numbers of notes per octave exist. The western European 7-tone and 12-tone scales simply reflect a melodic preference for intervals approximately the size of "whole and half steps."

A sensitive musical performer tends naturally to play harmonically in tune except where instrumental design makes this impossible, as with keyboard and percussion instruments, or where such refinements are not audible due to speed or complexity. Playing harmonically in tune (listening to other performers to adjust the pitch) results in small melodic interval displacements. While these variations in the pitch of theoretically "fixed" scale tones are melodically insignificant, their use reduces equal temperament to an abstraction. It is these precisions, adjusted *by ear,* which give clarity and resonance to simultaneously sounding tone combinations. Actual use of equal temperament in performance obscures this clarity by introducing a blurring element of dissonance into every musical interval except the octave.

A more harmful, if more subtle, effect of temperament is the inadequate conceptual model it presents to the composer. So long as musical usages are based upon a rational scale, the out-of-tuneness of temperament is a largely negligible consideration. But when many of these usages become outmoded and a search for new principles of musical organization begins, as has happened in the twentieth century, the one-sided model provided by equal temperament becomes a serious but largely unrecognized limitation. When musical organization based upon a linear (interval) scale replaces that based upon a ratio scale, there is a net loss in audible intelligibility. For instance, rhythmic order based merely upon additive equal increments is less "organic" than that based also upon the division of wholes into proportional parts, though in a given instance either kind of order may predominate. Ordering procedures based simply upon contour (that is, upon a fluctuation of greater and less without regard even for equal increments), and those based merely upon a perception of similarity and difference are still less integrative.[9]

The functioning of a higher level of organization does not negate the importance of lower levels, but subordinates them. If a lower level of scalar order is intended to predominate, then the tendency of a listener

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[8] Cf. James Murray Barbour, *Tuning and Temperament,* East Lansing, Michigan State College Press, 1951.

[9] Cf. Stevens, "On the Theory of Scales of Measurement." Stevens' *nominal* and *rank order* scales are here in question.

to read a higher level of order into the composition must be counteracted. For example, if *general* pitch contour is the main design element, then *precise* melodic interval patterns should be relatively hard to measure by ear. Similarly, if the composer intends simultaneous interval combinations to be heard as "vertical melody," then it should be made hard to hear these combinations as chords subordinated to roots: blend should be underemphasized, rhythmic displacement, timbre contrast, and dissonance perhaps emphasized. Many of the devices (especially the prohibitions) of serial and "atonal" composition are explicable most satisfactorily by reference to this principle. Linear ordering (by intervallic contour, motif, serial pattern) lends itself to much more elaborate systems than does merely qualitative ordering, but even these often have too little mnemonic power to integrate complex music so that its interrelations on a large time-scale are evident to the ear. Proportional organization has much more mnemonic power than this, but where pitch is concerned, western Europe tradition so far offers too limited a horizon on this front to satisfy contemporary aesthetic needs. This horizon is extensible, however. The rational pitch scale offers a hierarchy of relationships from the simplest (identity) to complexities beyond the analytical capabilities of human hearing. We are operating nowhere near these psychoacoustic limits at present. It is possible, with sufficient determination and effort, to extend rational intelligibility much farther into complex relationships.[10]

The most significant difference between proportional and linear organization is that the former makes possible the relation of all musical sounds to a *common reference point.* A group of pitches may be very complexly related to each other, but often all of them can be simply related to another pitch, which need not even be present. Thus, the missing pitch is strongly *implied* by the complex group. The *root* of a chord, the *tonic* of a tonality, the *principal tonality* of a modulating movement are all examples of this principle.

Before embarking upon a detailed derivation of pitch ratio scales, let us examine a parallel formal situation. Not only pitches but durations are proportionally relatable. A pattern of melodic rhythm has a "beat" if all its note-durations approximate simple proportions and can, therefore, be related to a single tempo.[11] In a contrapuntal ensemble, it may be that no two melodic rhythms ever coincide and that each melody has a different tempo and beat grouping. But in the ensemble, a single beat may emerge, which is the over-all tempo, the common denominator of all the different beats of the individual parts. Organization into periodic

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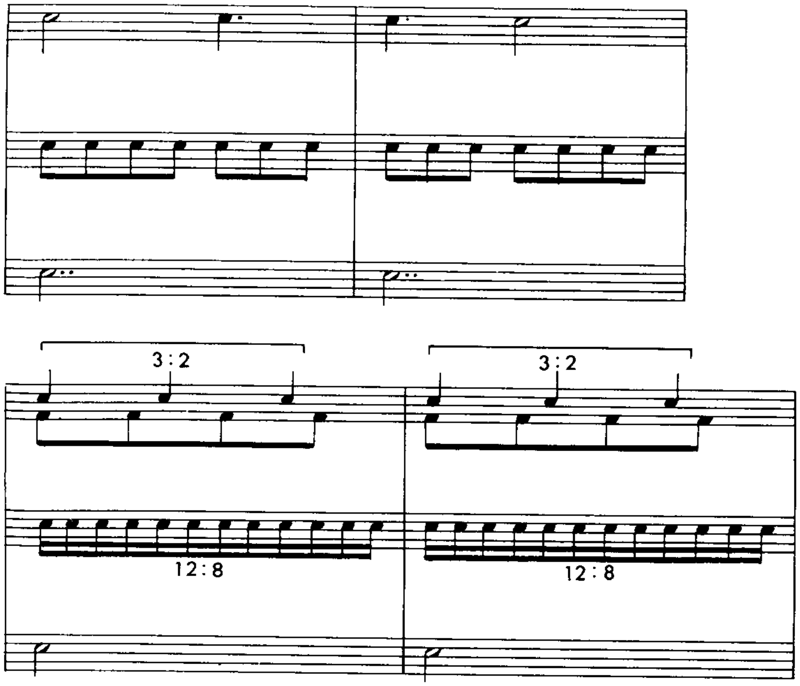
[10] To cite a single occidental example: Harry Partch's *Genesis of a Music,* Madison, University of Wisconsin Press, 1949.

[11] Cf. Erickson, *op. cit.*

rhythmic divisions often applies also to beat groupings, to measures, and to phrases.

Durational proportionality exists on a note-to-note scale, on a beat-to-beat scale (since beats can have proportions other than equality), on a measure-to-measure scale, on a phrase-to-phrase scale, on a section-to-section scale, and even on a movement-to-movement scale. The composer's problem is to provide a variety of durations whose proportions to each other and also to larger durations, of which they are part, are intelligible to a listener. These relationships aid greatly in creating either the sense of a completed whole or else of a significantly uncompleted one. A predominance in rhythm of the simplest ratios (equality: 1/1, duple subdivision: 2/1, and triple subdivision: 3/1, or multiples of these) makes for ease in intelligibility. So long as all proportions are referred to unity, there is a single tempo. Secondary durational proportions (such as 3/2 or 4/3) create over-all tempos both slower and faster than their member durations. These can result either from additive durations or from different simultaneous subdivisions of the same whole duration (Ex. 1).

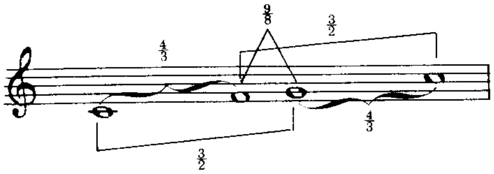
A linear scale of tempos exists in practice, but is not customarily described as such. Between any two given tempos there can be a scale



Ex. 1. The creation of faster and slower common tempos by both additive and divisive rhythms. By relating both 4 and 3 to 1, we obtain a periodic 7 meter. By relating both 3 and 4 to a larger 1, we obtain a periodic 12 meter.

of gradually accelerated tempos. The normal way to perform such an increase is by graduating "equal" increments of tempo between the two limiting tempos. Except for this technique of *accelerando* and *rallentando,* successive tempos are usually proportional or at least approximately so. There is no evidence that any perceptible interrelation exists between linear and proportional gradations of tempos. Tempo rubato can be described as fluctuation along this linear tempo scale.

Such an interrelation of linear and proportional relationships is of the essence in dealing with pitch scales, however, where it accounts for the conflict between melodic symmetry and harmonic simplicity. If we begin, in dealing with pitch, with the octave (2/1), cyclically dividing the pitch spectrum, the next question is how to divide each octave.[12] A desire for melodic symmetry suggests a division of the octave into two "equal" intervals. This would be the geometric mean ($\sqrt{2}$). But this interval, the equal-tempered tritone, is harmonically extremely dissonant. If, instead, the arithmetic mean is selected, a melodically unequal division into a 3/2 (perfect fifth) and a 4/3 (perfect fourth) results. If this arithmetic division is taken, starting with the upper pitch as well as with the lower, an interval of difference, 9/8 (the diatonic whole step, or major second) results.[13] It separates the two disjunct tetrachords (4/3 intervals) of the octave (Ex. 2).

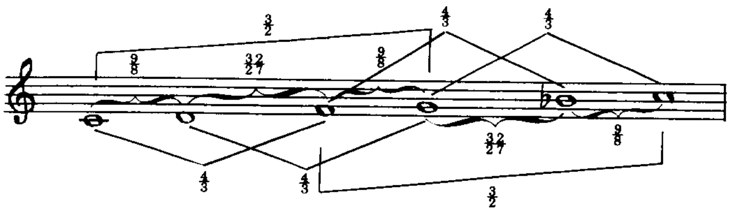


Ex. 2

[12] Cf. Alain Daniélou, *Traité de Musicologie, Comparée,* Paris, Hermann. (Cf. also Daniélou, *An Introduction to the Theory of Musical Scales,* London, The India Society.) Daniélou's approach to the subject of pitch scales is ethnic and historical. The present discussion is concerned only peripherally with these aspects of the subject. The scale derivations given below are not based upon a scholarly reconstruction of theories of Pythagoras, King Fang, ancient Sanskrit writers, or, for that matter, any other theorists. The application of these patterns of order in diverse musical systems of other times and places is a matter of great interest and importance, but it does not fall within the scope of the present inquiry. An attempt has been made, however, to lay the groundwork for an application of these scales within our own musical tradition and to present them in the perspective of a more general theory of scalar order. Divergences between this presentation of scale theory and Daniélou's are traceable mainly to a difference of musical cultural context. Although he is attempting to transcend any specific musical tradition in his *Traité,* and although in it he has considerably moderated the harsh criticism of occidental musical traditions which he included in his earlier book on the same subject, Daniélou's sympathies are obviously with Asiatic musical traditions and practices. This writer, on the contrary, while making some of the same criticisms, views the occidental tradition as potentially a great one, but with serious flaws which can nevertheless perhaps be corrected both in theory and practice.

[13] All tones are transposed by octave into the same pitch range.

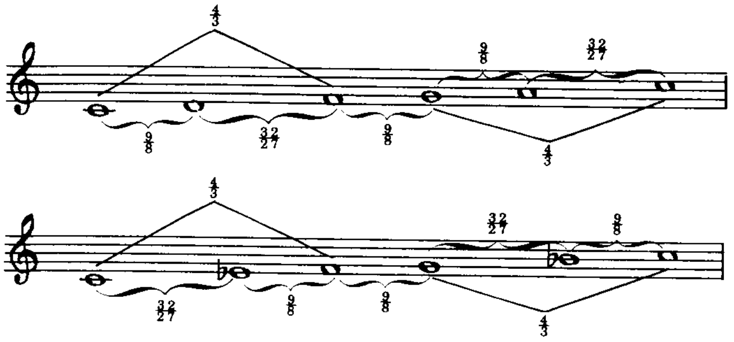
If the two fifths are divided similarly, a pentatonic scale results (Ex. 3).



Ex. 3

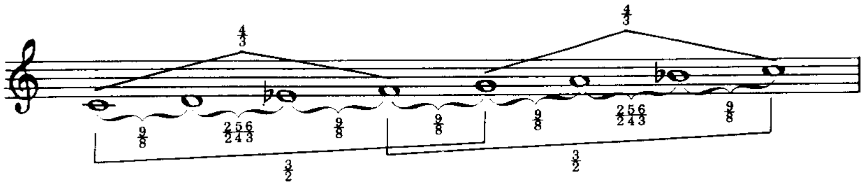
By the introduction of the new prime number, 3, the "cycle" of fifths as well as that of octaves is implied. If we take the lower pitch as 1/1 and the upper as 2/1, the arithmetic mean is 3/2. If we take the upper pitch as 1/1 and the lower as 1/2, the arithmetic mean is 2/3 which, raised an octave, is 4/3.

If instead the two tetrachords are similarly divided, either of two pentatonic scales results (Ex. 4).



Ex. 4

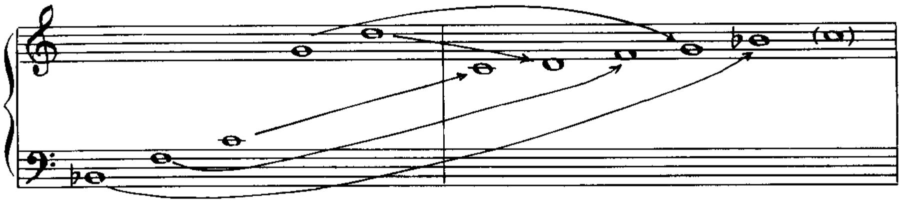
In all these cases the whole step is used to divide the tetrachords. The difference in size between the whole step and the minor third, the diatonic half step (256/243, or $2^8/3^5$), may be considered melodically negligible, in which case a pentatonic scale is melodically adequate. If this interval is considered melodically significant, a 7-tone scale can be derived (Ex. 5).



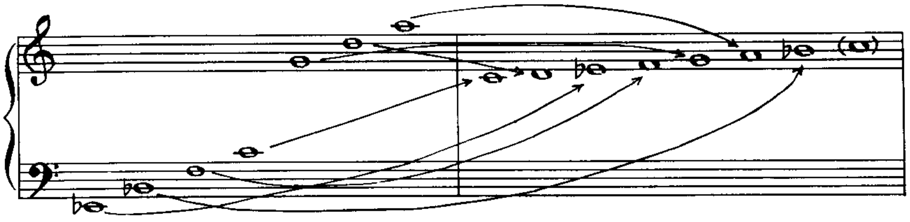
Ex. 5

Scales which are not modes of this same diatonic scale are obtainable by melodically inverting the two 32/27 intervals of a pentatonic scale.

Pythagorean 5-tone and 7-tone scales can be shown to be derived from cyclic tuning in perfect fifths (Exx. 6 and 7).

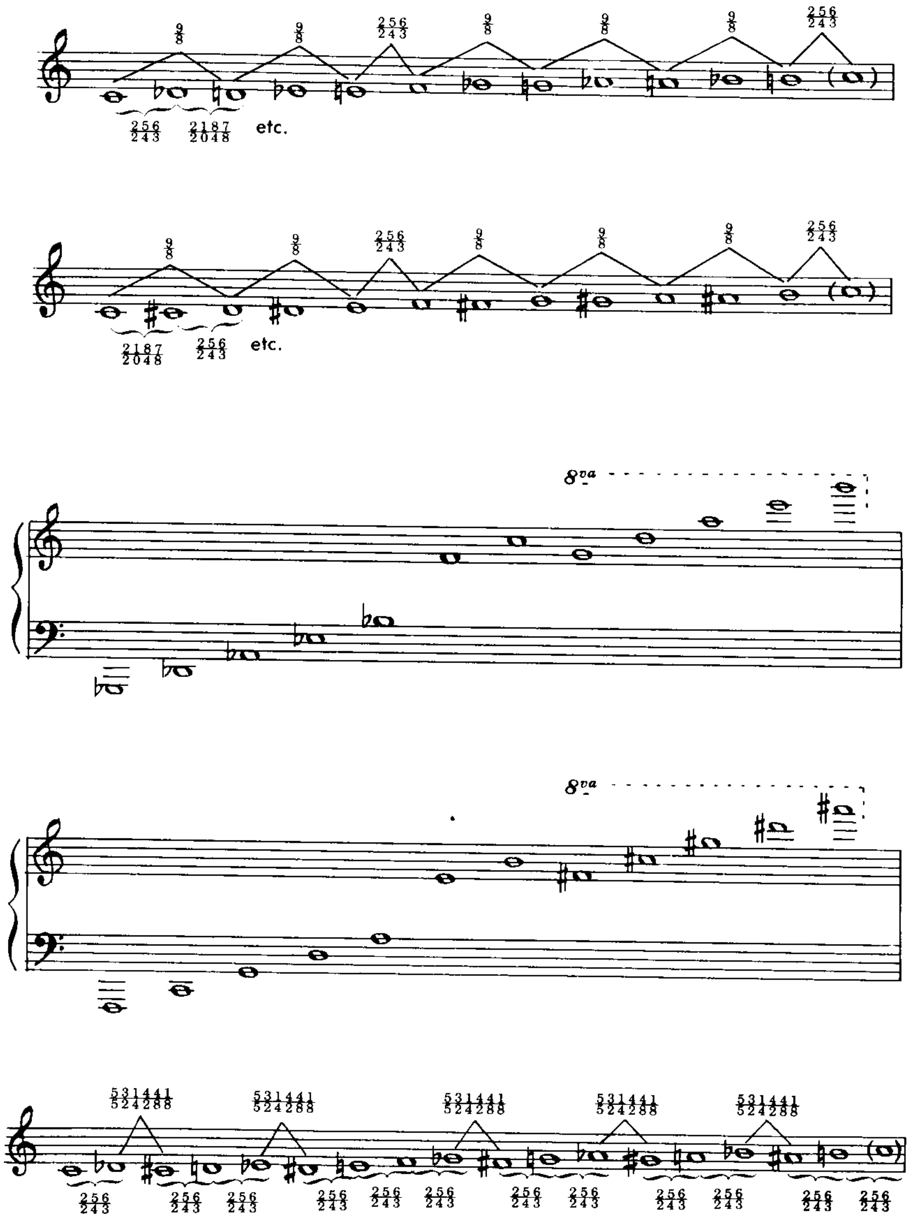


Ex. 6



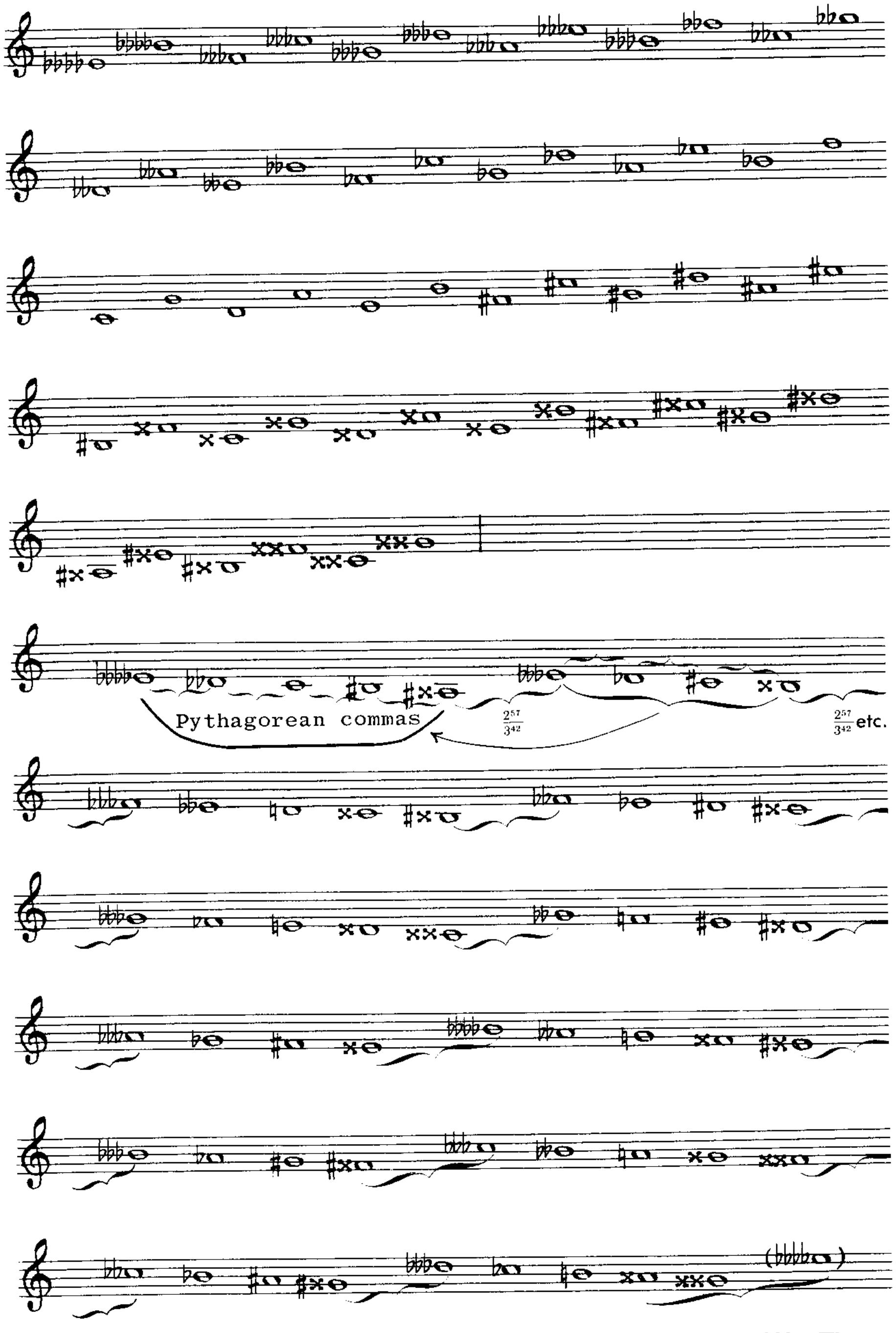
Ex. 7

Each whole step (9/8) can be divided (factored) into a diatonic half step (256/243) and a chromatic half step (2187/2048 or $3^7/2^{11}$). The chromatic half step can be placed either above (as a sharp) or below (as a flat) a diatonic scale tone. The enharmonic interval (between these two alternative possibilities) is called the Pythagorean comma. The Pythagorean 12-tone scale can also be derived from cyclic tuning in perfect fifths (Ex. 8).



Ex. 8. Two derivations by cyclic tuning in perfect fifths of the 12-tone (chromatic) scale and their combination into a 17-tone enharmonic scale entirely composed of diatonic half steps (256/243) and enharmonic Pythagorean commas (531441/524288 or $3^{12}/2^{19}$).

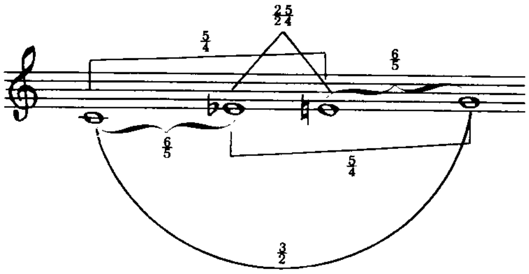
If cyclic tuning in perfect fifths is extended further, enharmonic equivalents are multiplied until we obtain a 53-tone scale, all but eleven of whose adjacent intervals are Pythagorean commas (Ex. 9).



Ex. 9. Derivation of a 53-tone enharmonic scale, by cyclic tuning in perfect fifths. The next-to-last tone is higher than the last by approximately one-fourth of a Pythagorean comma. Except for this tone, the scale is given in ascending order of pitch. The bracketed intervals ($2^{57}/3^{42}$) are approximately three-fourths the size of a Pythagorean comma. Derivation is carried out as far as possible without overlapping enharmonic regions.

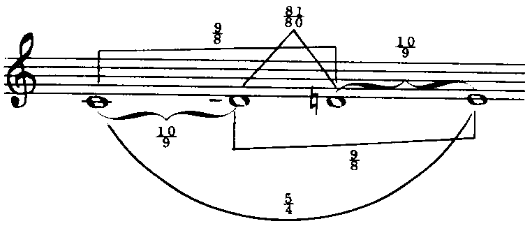
The Pythagorean scale provides a very limited amount of consonance. Only its perfect octaves, fifths, and fourths and its major seconds and minor sevenths have no near interval equivalents which are more consonant. If the first principles used in dividing the octave are extended further, a system of scales parallel to the Pythagorean system is possible, providing many near interval equivalents which yield a much greater number of consonances.

If the arithmetic mean is taken as before, this time within the perfect fifth (3/2), the major and the minor thirds in "just intonation" (5/4 and 6/5) are obtained. The chromatic half step (25/24) is the interval of difference (Ex. 10).



Ex. 10

Similarly, if the arithmetic mean is taken within the major third, the two major whole tones of just intonation, sometimes called the major and the minor tone (9/8 and 10/9), are obtained. The diatonic comma or comma of Didymus (81/80) is the interval of difference (Ex. 11).[14]



Ex. 11

Just tuning, involving ratios based on prime numbers 1, 2, 3, and 5 (rather than on 1, 2, and 3 only, as in Pythagorean tuning), does not readily yield pentatonic scales but is extremely useful for heptatonic scales (Ex. 12).

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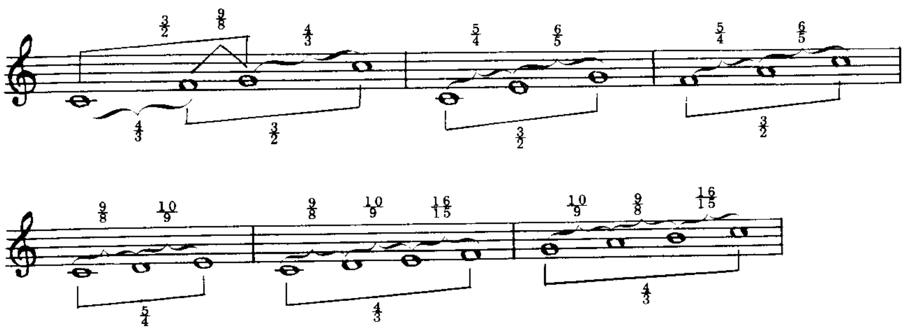
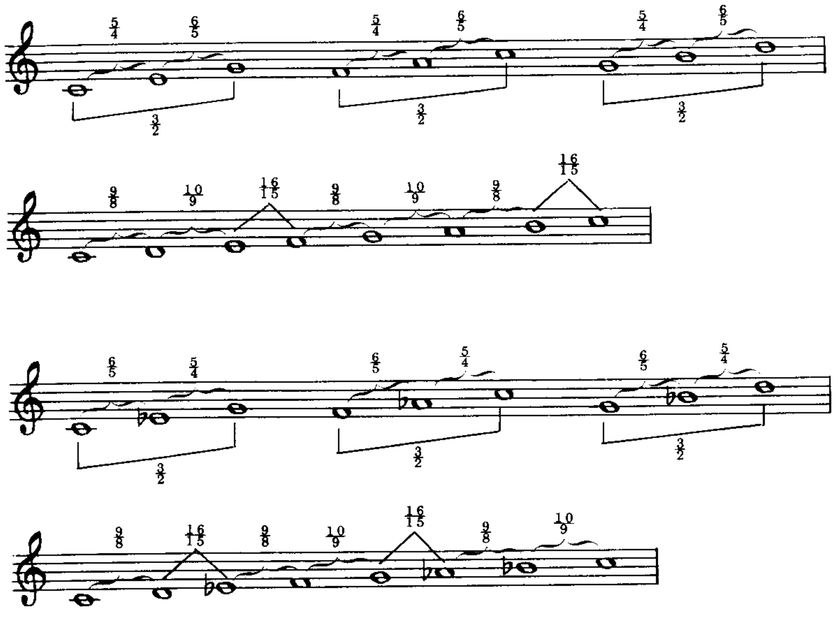
[14] + means raised by a diatonic comma; – means lowered by that interval.

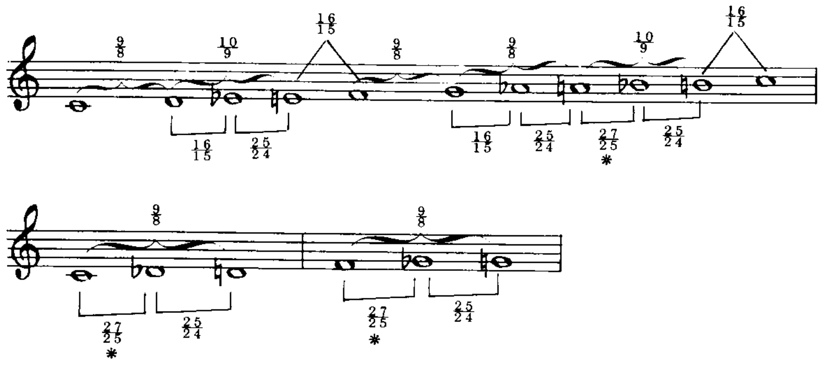


Ex. 12. Using both arithmetic means of the octave (2:1), and the higher arithmetic mean of the two overlapping perfect fifths and of the major third in the lower fifth, a nearly complete 7-tone scale is obtained. If, in addition the two tetrachords are divided analogously (except for the order of the two whole tones), the 7-tone scale results.

The harmonic basis of just intonation is the major triad (4:5:6, which expresses the arithmetic division of the perfect fifth). The inversion of this is the minor triad. The arithmetic division within major thirds can be shown to result from triadic tuning (Exx. 13 and 14).

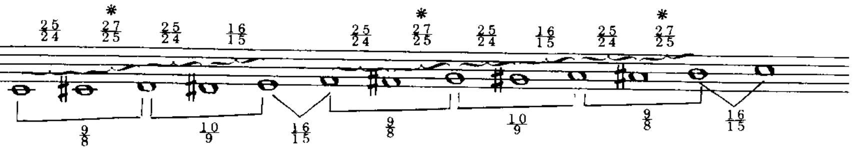


Ex. 13. Triadic derivation of the 7-tone major and minor scales (in just intonation). These two scales are modally related: that is, they have the same interval pattern, but from different starting points.



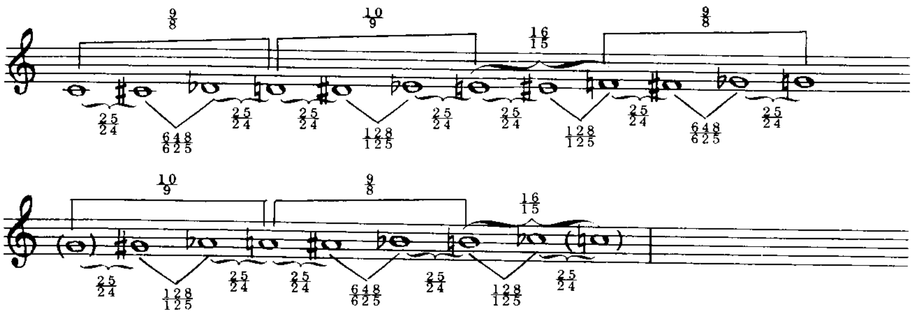
Ex. 14. Mixture of the two scales derived in Ex. 13. The two undivided whole tones are divided in a manner analogous to the other whole tones, yielding a 12-tone scale.

A just intonation twelve-tone scale can be based upon a mixture of these two diatonic scales (Ex. 15).



Ex. 15

If these two scales are combined, enharmonic scales result. Between E and F either E♯ or F♭ can be added. Similarly between B and C, either B♯ or C♭ can be added (Ex. 16).



Ex. 16. Mixture of the scales derived in Exx. 14 and 15 into an enharmonic 19-tone scale. The 648/625 and 128/125 intervals are approximately equal to quarter tones. The difference between them is a diatonic comma (81/80), which is the difference in size between the major and the minor whole tones.

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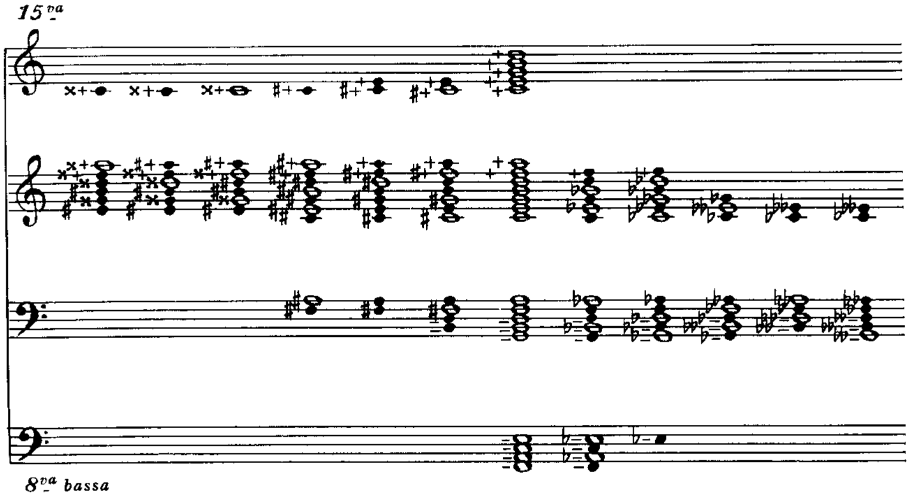
*[15] The flat (♭) lowers by a chromatic half step (25/24); the sharp (♯) raises by the same interval.

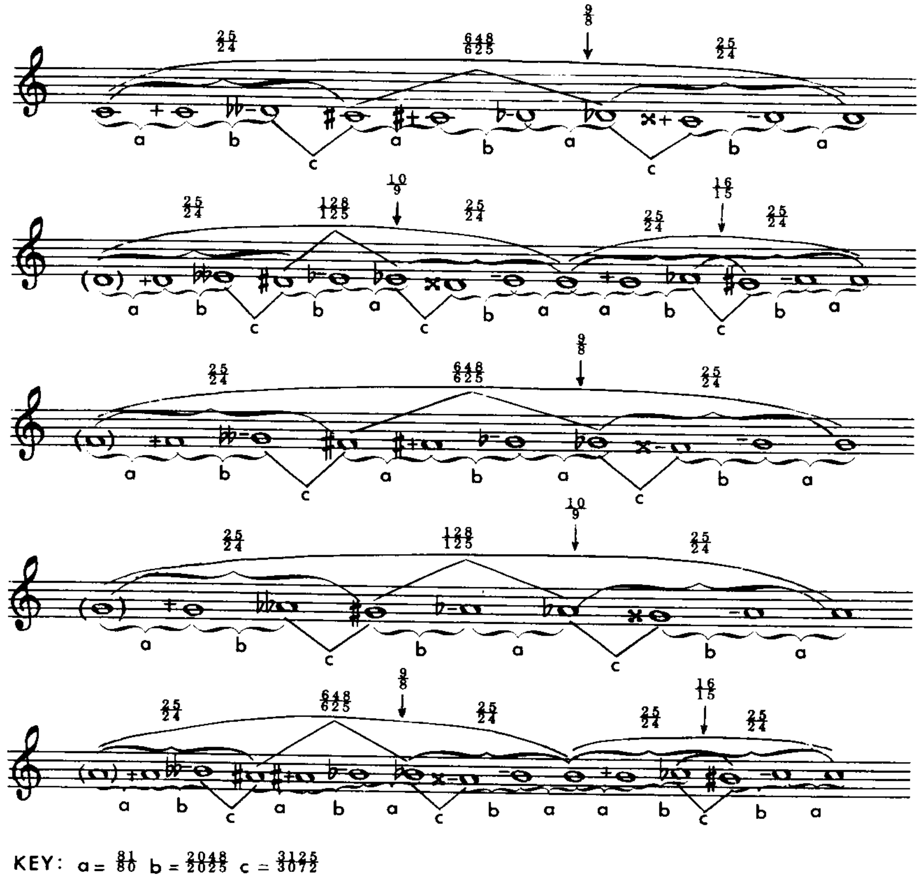
If an interlocking system of triads is set up, such that every tone participates in four different capacities (as root, as fifth, as major third, and as minor third), and if this derivation is stopped at points where the twelve chromatic regions of the octave would overlap, a 53-tone scale results.[16] (Exx. 17 and 18).



Ex. 17 Key: a = 81/80 b = 2048/2025 c = 3125/3072

81/80 and 2048/2025 are very close to the same size, but 3125/3072 is almost twice as large. If we add to the scale one of each of these pairs: D♭♭ or C♯−, D♭+ or C𝄪, E♭♭+ or D♯−, E♭+ or D𝄪−, F♭+ or E♯−, G♭♭ or F♯−, G♭+ or F𝄪, A♭♭+ or G♯−, A♭+ or G𝄪−, B♭♭ or A♯−, B♭+ or A𝄪, and C♭+ or B♯−, a 65-tone scale of almost equal intervals results. Each 3125/3072 is thereby divided into an 81/80 and a 15625/15552 or $5^6/3^5 \cdot 2^6$ (which is somewhat larger than half the size of an 81/80). If both of each pair were chosen, the chromatic regions would overlap (see below). Note that the *diatonic* regions overlap not only because of 𝄪's and ♭♭'s, but also at the intervals F♭, E♯ and C♭, B♯. The choice between these alternative notes is clear from the triadic derivation, except in the case of the F♭+, E♯− choice and the C♭+, B♯− choice. All these additions to the scale, however, spoil the regularity of the pattern of adjacent intervals within the larger diatonic intervals.

[16] Derivation of a just intonation 53-tone enharmonic scale, showing the diatonic and chromatic scales contained within it. The pattern of adjacent intervals within each major whole tone, that within each minor whole tone, and that within each diatonic semitone is the same. There are only three kinds of adjacent intervals: 81/80, the diatonic comma; 2048/2025, which is a diatonic comma smaller than 128/125, the smaller of the two enharmonic intervals; and 3125/3072, which separates the twelve chromatic regions of the octave.



Ex. 18

A comparison of these recurrent patterns with the whole and half-step pattern of the 7-tone diatonic major scale is interesting. If the 9/8 is called *A*, 10/9 *B*, and 16/15 *C*, the 7-tone pattern is *ABCABAC*, where *A* and *B* are near the same size and *C* is not. Within the 9/8, if the 81/80 is called *A*, the 2048/2025 *B*, and the 3125/3072 *C*, the pattern is *ABCABACBA*, where *A* and *B* are near the same size and *C* is not. Within the 10/9, the pattern is *ABCBACBA*. Within the 16/15, it is *ABCBA*. In all cases, the *C* intervals are the first place where chromatic regional overlapping occurs in further derivation.

The triadic system of this derivation can theoretically be extended as far as desired, but the linear (melodic) intervals of adjacent notes of the resulting scale, at points where chromatic regions overlap, are at the largest only 2 cents wide. This interval (32805/32768) is the difference in size between the 81/80 and the 2048/2025. Its size is impractically close to the psychoacoustical threshold of linear pitch discrimination. By

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C |  | C+ |  | D♭♭− |  | C♯ |  | C♯+ |  | D♭− |  | D♭ |  | Cx+ |  | D− |  | D |  | D+ |  | E♭♭ |  | D♯ |  | E♭− |  | E♭ |  | Dx |  | E− |
| 0 |  | 5 |  | 10 |  | 18 |  | 23 |  | 28 |  | 33 |  | 41 |  | 46 |  | 51 |  | 56 |  | 61 |  | 69 |  | 74 |  | 79 |  | 87 |  | 92 |
| DO |  | + |  | ++ |  | ♯ |  | L− |  | L+ |  | ♭ |  | −− |  | − |  | RE |  | + |  | ++ |  | ♯ |  | L |  | ♭ |  | −− |  | − |
| $\frac{1}{1}$ |  | $\frac{81}{80}$ |  | $\frac{128}{125}$ |  | $\frac{25}{24}$ |  | $\frac{135}{128}$ |  | $\frac{16}{15}$ |  | $\frac{27}{25}$ |  | $\frac{1125}{1024}$ |  | $\frac{10}{9}$ |  | $\frac{9}{8}$ |  | $\frac{729}{640}$ |  | $\frac{144}{125}$ |  | $\frac{75}{64}$ |  | $\frac{32}{27}$ |  | $\frac{6}{5}$ |  | $\frac{625}{512}$ |  | $\frac{100}{81}$ |
|  | AB |  | A |  | ABBC |  | AB |  | A |  | AB |  | ABBC |  | A |  | AB |  | AB |  | A |  | ABBC |  | A |  | AB |  | ABBC |  | A |  |
|  | 5 |  | 5 |  | 8 |  | 5 |  | 5 |  | 5 |  | 8 |  | 5 |  | 5 |  | 5 |  | 5 |  | 8 |  | 5 |  | 5 |  | 8 |  | 5 |  |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| (E−) |  | E |  | E+ |  | F♭ |  | E♯ |  | F− |  | F |  | F+ |  | G♭♭− |  | F♯ |  | F♯+ |  | G♭− |  | G♭ |  | Fx+ |  | G− |  | G |  | G+ |  | A♭♭ |  | G♯ |  | A♭− |  | A♭ |  | Gx |  | A− |
| (92) |  | 97 |  | 102 |  | 107 |  | 115 |  | 120 |  | 125 |  | 130 |  | 135 |  | 143 |  | 148 |  | 153 |  | 158 |  | 166 |  | 171 |  | 176 |  | 181 |  | 186 |  | 194 |  | 199 |  | 204 |  | 212 |  | 217 |
| (−) |  | MI |  | + |  | ++ |  | −− |  | − |  | FA |  | + |  | ++ |  | ♯ |  | L− |  | L+ |  | ♭ |  | −− |  | − |  | SOL |  | + |  | ++ |  | ♯ |  | L |  | ♭ |  | −− |  | − |
| ($\frac{100}{81}$) |  | $\frac{5}{4}$ |  | $\frac{81}{64}$ |  | $\frac{32}{25}$ |  | $\frac{125}{96}$ |  | $\frac{320}{243}$ |  | $\frac{4}{3}$ |  | $\frac{27}{20}$ |  | $\frac{512}{375}$ |  | $\frac{25}{18}$ |  | $\frac{45}{32}$ |  | $\frac{64}{45}$ |  | $\frac{36}{25}$ |  | $\frac{375}{256}$ |  | $\frac{40}{27}$ |  | $\frac{3}{2}$ |  | $\frac{243}{160}$ |  | $\frac{192}{125}$ |  | $\frac{25}{16}$ |  | $\frac{128}{81}$ |  | $\frac{8}{5}$ |  | $\frac{625}{384}$ |  | $\frac{400}{243}$ |
|  | AB |  | AB |  | A |  | ABBC |  | A |  | AB |  | AB |  | A |  | ABBC |  | AB |  | A |  | AB |  | ABBC |  | A |  | AB |  | AB |  | A |  | ABBC |  | A |  | AB |  | ABBC |  | A |  |
|  | 5 |  | 5 |  | 5 |  | 8 |  | 5 |  | 5 |  | 5 |  | 5 |  | 8 |  | 5 |  | 5 |  | 5 |  | 8 |  | 5 |  | 5 |  | 5 |  | 5 |  | 8 |  | 5 |  | 5 |  | 8 |  | 5 |  |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| (A−) |  | A |  | A+ |  | B♭♭− |  | A♯ |  | A♯+ |  | B♭− |  | B♭ |  | Ax+ |  | B− |  | B |  | B+ |  | C♭ |  | B♯ |  | C− |  | C |
| (217) |  | 222 |  | 227 |  | 232 |  | 240 |  | 245 |  | 250 |  | 255 |  | 263 |  | 268 |  | 273 |  | 278 |  | 283 |  | 291 |  | 296 |  | 301 |
| (−) |  | LA |  | + |  | ++ |  | ♯ |  | L− |  | L+ |  | ♭ |  | −− |  | − |  | SI |  | + |  | ++ |  | −− |  | − |  | DO |
| ($\frac{400}{243}$) |  | $\frac{5}{3}$ |  | $\frac{27}{16}$ |  | $\frac{128}{75}$ |  | $\frac{125}{72}$ |  | $\frac{225}{128}$ |  | $\frac{16}{9}$ |  | $\frac{9}{5}$ |  | $\frac{1875}{1024}$ |  | $\frac{30}{27}$ |  | $\frac{15}{8}$ |  | $\frac{243}{128}$ |  | $\frac{48}{25}$ |  | $\frac{125}{64}$ |  | $\frac{160}{81}$ |  | $\frac{2}{1}$ |
|  | AB |  | AB |  | A |  | ABBC |  | AB |  | A |  | AB |  | ABBC |  | A |  | AB |  | AB |  | A |  | ABBC |  | A |  | AB |  |
|  | 5 |  | 5 |  | 5 |  | 8 |  | 5 |  | 5 |  | 5 |  | 8 |  | 5 |  | 5 |  | 5 |  | 5 |  | 8 |  | 5 |  | 5 |  |

KEY: A = $\frac{2048}{2025}$ B = $\frac{32805}{32768}$ C = $\frac{1600000}{1574323}$ ∴AB = $\frac{81}{80}$ ∴BC = $\frac{15625}{15552}$ ∴ABBC = $\frac{3125}{3072}$

$(A > C > B; A > BC)$

♯ = $\times\frac{25}{24}$ x = $\times(\frac{25}{24})^2$ ♭ = $\div\frac{25}{24}$ + = $\times\frac{81}{80}$ − = $\div\frac{81}{80}$

The numerals above and below (not the ratios) give approximate interval sizes in *savarts* (*circa.* 301 per octave).

The symbols associated with the DO, RE, MI, etc. are from Alain Daniélou's *Introduction to the Theory of Musical Scales.*

Note particularly the 7-tone diatonic scale tones, which are the only tones with AB ($\frac{81}{80}$) intervals on each side. Also note that the ABBC ($\frac{3125}{3072}$) intervals delineate the borderlines between the twelve pitch regions.

The 53-tone just intonation scale showing the ratios of each tone to the tonic (C).

contrast, in most cases a discrepancy of 2 cents in a *harmonically* played perfect octave, fifth, or fourth is easy to hear. Melodic intervals as small as diatonic commas (as in the 53- or even the 65-tone scale) are not difficult to discriminate.

Each prime number used in deriving a harmonic scale contributes a characteristic psychoacoustical meaning. One to one is the relationship of identity; two to one, of recurrence or repetition. Three to one (or three to two) contributes polarity: a sense of gravity, of right-side-up and upside-down (e.g. the root-fifth relationship and the tonic-dominant relationship).[17] Five, in combination with the other prime relationships, contributes major-minor coloration.

In a manner analagous to the scale derivations given above, other prime numbers can be introduced into rational pitch scales.[18] There would be no aesthetic point in introducing a new prime into harmonic usage unless its psychoacoustical meaning were quite distinct. To hazard a guess based on *consonant* use of the dominant 7th chord (tuned in the ratio 4:5:6:7), prime number 7 may be said to contribute a sense of centralized instability, suspending the dominant-tonic (3 to 2) polarity. 7/6 and 8/7 are the two arithmetic means of the perfect 4th (4/3) (derived as before). Their harmonic use provides a consonant tritone (7/5), a consonant minor 7th (7/4) and a consonant major second (8/7).

Interval-scale thinking emphasizes symmetry of design. The harmonic and tonal meaning of symmetrical pitch structures is *ambiguity*. Chordally they produce either a sense of multiple root possibilities or of no satisfactory root possibility. Tonally they cause either a sense of several possible tonics or of no adequate tonic.

Ratio-scale thinking, on the contrary, emphasizes a hierarchical subordination of details to the whole or to common reference points. The harmonic and tonal meaning of proportional pitch structures is clarity and a sense of direction. Symmetrically repeating rational pitch intervals does not bring about a circular return to the original pitch, but progressively changing spiral approximations of it (cf. the "circle" of fifths; also the process of harmonic displacement by diatonic comma in triadic music). So long as music is designed by principles based upon ratio-scale order, distortion of its pitch proportions, whether by equal-temperament or simply by imperfect performance, does not destroy its psychological effect of "progression" and change. Music designed by symmetrical

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[17] A pitch carries with it by implication, if not in acoustical fact, a system of overtones standing in the relation of a harmonic series to the fundamental (taken as unity). Thus if two pitches have the relation 3:1, the entire harmonic series of the higher pitch is contained in that of the lower one, while the reverse is not the case. Hence, the perfect fifth is traditionally consonant, while the perfect fourth is not. Hence also, dominant to tonic is a restful progression, since the new root contains the harmonic series of the former one entirely.

[18] Cf. Partch, *op. cit.*

principles systematically based upon an equal-interval scale (e.g. serial organization of pitch) does tend to destroy this effect.

In our time, the resources of ratio-scale pitch organization have come to seem exhausted and outmoded. Thus there has been a deliberate reliance upon less sophisticated organizational techniques: interval-scale, ordinal-scale, and nominal-scale methods of creating musical order. This research has produced, by implication, new orders of complexity in ratio-scale ordering, since we cannot entirely suspend the action of our intelligences on this level so as to attend exclusively to other kinds of order. In practice, we already use much more complex pitch ratios than are provided in traditional tonality. In order to make intelligible this implied complexity, an extension of ratio-scale techniques is needed. Typically, theory lags behind practice.

The "emancipation of the dissonance"[19] did not solve the problem of harmonic freedom. While rendering permanently obsolete the old black-and-white division into consonant and dissonant, it in no sense abolished the tonal hierarchy. On the contrary, it has made more urgent than ever the need for more basic principles of tonal order to integrate ever more complex pitch situations. Serial technique is only an interim solution. Its contributions to musical intelligibility and expressivity need to be subsumed under a larger and less restrictive whole.

Traditional tonality depends upon ratio-scale ordering: upon harmonic relations between chord-roots, tonal centers, and principal tonics. Likewise, classical metricality and symmetrical phrasing depend upon proportional use of durations and tempos. Both these techniques are capable of very great extension beyond present general usage,[20] while still remaining audibly intelligible and expressive. There is no need to sacrifice the linear ordering methods of serial technique in order to achieve this. What must be given up is the limitation of equal temperament. The ordering potential of ratio scales includes that of linear scales, but not vice versa.

The adoption of a microtonal ratio scale does not imply a return to modal monophony. Such a scale is equally applicable to harmonic polyphony. The fact that our instruments are designed for equal temperament is not an insuperable obstacle. Except in the case of keyboard and percussion instruments with fixed tuning, sufficient control of the pitch by the performer can at present be expected. The new possibilities opened up by electronic instruments and electronic synthesis already begin to provide more flexible control of pitch than has hitherto been possible.

[19] Arnold Schoenberg's term: cf. *Style and Idea,* New York, Philosophical Library, 1950.

[20] The mature works of Elliott Carter, to cite one outstanding example, extend rhythmic technique in this direction.

Great complexity in the materials of art calls for a compositional technique with maximum organizing power. To establish connection between the known and rational and familiar, on the one hand, and the unknown and irrational and unpredictable on the other, requires subjecting them to the same measure. Proportionality is such a common measure, if we bear in mind the modifying principles of variation and approximation. It is not incompatible with other modes of organization, such as serial ordering. It applies with equal effectiveness to formal, rhythmic, and pitch organization. It can be realized best by ear, in the case of pitch; by kinesthetic perception, in the case of rhythm; and by intuitive timing, in the case of formal divisions. Yet it is capable of intellectual formulation and manipulation. Most important of all, such a technique reestablishes a connection which has been broken, a connection with ancient and worldwide traditions of aesthetic order.